UNIT-V ::

Frequency domain Analysis: Introduction- frequency domain specifications- polar plots- bode plots- Nyquist stability criterion

....

The response of a system for the sinusoidal input is called sinusoidal response. The ratio of sinusoidal response and sinusoidal input is called ***sinusoidal transfer function*** of the system and in general, it is denoted by $T(j\omega)$. The sinusoidal transfer function is the frequency domain representation of the system, and so it is also called ***frequency domain transfer function.***

The frequency response can be evaluated for open loop system and closed loop system. The frequency domain transfer function of open loop and closed loop systems can be obtained from the s-domain transfer function by replacing s by $j\omega$ shown below.

Open loop transfer function : $G(s) \xrightarrow{s=j\omega} G(j\omega) = |G(j\omega)| \angle G(j\omega)$ .....(3.1)

Loop transfer function : $G(s)H(s) \xrightarrow{s=j\omega} G(j\omega)H(j\omega) = |G(j\omega)H(j\omega)| \angle G(j\omega)H(j\omega)$ .....(3.2)

Closed loop transfer function: $M(s) \xrightarrow{s=j\omega} M(j\omega) = |M(j\omega)| \angle M(j\omega)$ .....(3.3)

where, $|G(j\omega)|$, $|M(j\omega)|$, $|G(j\omega)H(j\omega)|$ are Magnitude functions

$\angle G(j\omega)$, $\angle M(j\omega)$, $\angle G(j\omega)H(j\omega)$ are Phase functions.

Note : *For unity feedback system, H(s) = 1 and open loop and loop transfer functions are same.*

The advantages of frequency response analysis are the following.

1. The absolute and relative stability of the closed loop system can be estimated from the knowledge of their open loop frequency response.
2. The practical testing of systems can be easily carried with available sinusoidal signal generators and precise measurement equipments .
3. The transfer function of complicated systems can be determined experimentally by frequency response tests.
4. The design and parameter adjustment of the open loop transfer function of a system for specified closed loop performance is carried out more easily in frequency domain.
5. When the system is designed by use of the frequency response analysis, the effects of noise disturbance and parameters variations are relatively easy to visualize and incorporate corrective measures.
6. The frequency response analysis and designs can be extended to certain nonlinear control systems.

FREQUENCY DOMAIN SPECIFICATIONS

The performance and characteristics of a system in frequency domain are measured in terms of frequency domain specifications. The requirements of a system to be designed are usually specified in terms of these specifications.

The frequency domain specifications are,

1. Resonant peak , M_r
2. Resonant Frequency , ω_r
3. Bandwidth, ω_b
4. Cut-off rate
5. Gain margin, K_g
6. Phase margin, γ

Resonant Peak (M_r)

The maximum value of the magnitude of closed loop transfer function is called the resonant peak, M_r. A large resonant peak corresponds to a large overshoot in transient response.

Resonant Frequency (ω_r)

The frequency at which the resonant peak occurs is called resonant frequency, ω_r. This is related to the frequency of oscillation in the step response and thus it is indicative of the speed of transient response.

Bandwidth (ω_b)

The Bandwidth is the range of frequencies for which normalized gain of the system is more than –3 db. The frequency at which the gain is –3 db is called cut-off frequency. Bandwidth is usually defined for closed loop system

A large bandwidth corresponds to a small rise time or fast response.

Cut-off Rate

The slope of the log-magnitude curve near the cut off frequency is called cut-off rate. The cut -off rate indicates the ability of the system to distinguish the signal from noise.

Gain Margin ,K_g

The gain margin, K_g is defined as the value of gain, to be added to system, in order to bring the system to the verge of instability.

The gain margin, K_g is given by the reciprocal of the magnitude of open loop transfer function at phase cross over frequency. The frequency at which the phase of open loop transfer function is 180° is called the phase cross-over frequency, ω_{pc}.

$$\text{Gain Margin, } K_g = \frac{1}{|G(j\omega_{pc})|}$$

The gain margin in db can be expressed as,

$$K_g \text{ in db} = 20 \log K_g = 20 \log \frac{1}{|G(j\omega_{pc})|}$$

Phase Margin (γ)

The phase margin γ, is defined as the additional phase lag to be added at the gain cross over frequency in order to bring the system to the verge of instability. The gain cross over frequency ω_{gc} is the frequency at which the magnitude of the open loop transfer function is unity (or it is the frequency at which the db magnitude is zero).

The phase margin γ, is obtained by adding 180° to the phase angle ϕ of the open loop transfer function at the gain cross over frequency

$$\text{Phase margin, } \gamma = 180° + \phi_{gc},$$

$$\text{where, } \phi_{gc} = \angle G(j\omega_{gc})$$

FREQUENCY DOMAIN SPECIFICATIONS OF SECOND ORDER SYSTEM

RESONANT PEAK (M_r)

Consider the closed loop transfer function of second order system,

$$\frac{C(s)}{R(s)} = M(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The sinusoidal transfer function $M(j\omega)$ is obtained by letting $s = j\omega$.

$$\therefore M(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2}$$

$$= \frac{\omega_n^2}{-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2} = \frac{\omega_n^2}{\omega_n^2\left(-\frac{\omega^2}{\omega_n^2} + j2\zeta\frac{\omega}{\omega_n} + 1\right)} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\frac{\omega}{\omega_n}}$$

Let, Normalized frequency, $u = \left(\frac{\omega}{\omega_n}\right)$

$$\therefore M(j\omega) = \frac{1}{(1-u^2) + j2\zeta u}$$

Let, M = Magnitude of closed loop transfer function

α = Phase of closed loop transfer function.

$$M = |M(j\omega| = \left[\frac{1}{(1-u^2)^2 + (2\zeta u)^2}\right]^{\frac{1}{2}} = \left[(1-u^2)^2 + 4\zeta^2 u^2\right]^{-\frac{1}{2}}$$

$$\alpha = \angle M(j\omega) = -\tan^{-1}\frac{2\zeta u}{1-u^2}$$

The resonant peak is the maximum value of M. The condition for maximum value of M can be obtained by differentiating the equation of M with respect to u and letting dM/du = 0 when $u = u_r$,

where, $u_r = \frac{\omega_r}{\omega}$ = Normalized resonant frequency.

$$\frac{dM}{du} = \frac{d}{du}\left[(1-u^2)^2 + 4\zeta^2 u^2\right]^{-\frac{1}{2}} = -\frac{1}{2}\left[(1-u^2)^2 + 4\zeta^2 u^2\right]^{-\frac{3}{2}}\left[2(1-u^2)\,(-2u) + 8\zeta^2 u\right]$$

$$= \frac{-\left[-4u\,(1-u^2) + 8\zeta^2 u\right]}{2\left[(1-u^2)^2 + 4\zeta^2 u^2\right]^{\frac{3}{2}}} = \frac{4u\,(1-u)^2 - 8\zeta^2 u}{2\left[(1-u^2)^2 + 4\zeta^2 u^2\right]^{\frac{3}{2}}}$$

Replace u by u_r in equation and equate to zero.

$$\frac{4u_r\,(1-u_r^2)-8\zeta^2 u_r}{2\left[(1-u_r^2)^2+4\zeta^2 u_r^2\right]^{\frac{3}{2}}}=0$$

$$4u_r(1-u_r^2)-8\zeta^2 u_r=0 \quad \Rightarrow \quad 4u_r-4u_r^3-8\zeta^2 u_r=0$$

$$\therefore\ 4u_r^3=4u_r-8\zeta^2 u_r \quad \Rightarrow \quad u_r^2=1-2\zeta^2 \quad \Rightarrow \quad u_r=\sqrt{1-2\zeta^2}$$

Therefore, the resonant peak occurs when $u_r=\sqrt{1-2\zeta^2}$

Put this condition in the equation for M and solve for M_r.

$$\therefore M_r=\left.\frac{1}{\left[(1-u^2)^2+4\zeta^2u^2\right]^{\frac{1}{2}}}\right|_{u=u_r}=\frac{1}{\left[(1-u_r^2)^2+4\zeta^2u_r^2\right]^{\frac{1}{2}}}=\frac{1}{\left[(1-(1-2\zeta^2))^2+4\zeta^2(1-2\zeta^2)\right]^{\frac{1}{2}}}$$

$$=\frac{1}{\left[4\zeta^4+4\zeta^2-8\zeta^4\right]^{\frac{1}{2}}}=\frac{1}{\left[4\zeta^2-4\zeta^4\right]^{\frac{1}{2}}}=\frac{1}{\left[4\zeta^2(1-\zeta^2)\right]^{\frac{1}{2}}}=\frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\therefore \boxed{\text{Resonant peak, } M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$$

RESONANT FREQUENCY (ω_r)

Normalized resonant frequency, $u_r = \dfrac{\omega_r}{\omega_n} = \sqrt{1-2\zeta^2}$

The resonant frequency, $\omega_r = \omega_n\sqrt{1-2\zeta^2}$

BANDWIDTH (ω_b)

Let, Normalized bandwidth, $u_b = \dfrac{\omega_b}{\omega_n}$

When $u = u_b$, the magnitude M, of the closed loop system is $1/\sqrt{2}$ (or –3db).

Hence in the equation for M (equation 3.9), put $u = u_b$ and equate to $1/\sqrt{2}$.

$$\therefore\ M = \frac{1}{\left[(1-u_b^2)^2 + 4\zeta^2 u_b^2\right]^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}$$

On squaring and cross multiplying we get,

$$(1-u_b^2)^2+4\zeta^2 u_b^2 = 2 \quad \Rightarrow \quad 1+u_b^4-2u_b^2+4\zeta^2 u_b^2=2 \quad \Rightarrow \quad u_b^4-2u_b^2\,(1-\zeta^2)-1 = 0$$

Let, $x=u_b^2$; $\quad \therefore x^2-2(1-2\zeta^2)x-1=0$

$$\therefore x=\frac{2(1-2\zeta^2)\pm\sqrt{4(1-2\zeta^2)^2+4}}{2}=\frac{2(1-2\zeta^2)\pm 2\sqrt{(1+4\zeta^4-4\zeta^2)+1}}{2}$$

Let us take only the positive sign,

$$\therefore x=1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4}$$

But, $u_b=\sqrt{x}$; $\quad \therefore u_b=\sqrt{x}=\left[1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4}\right]^{\frac{1}{2}}$; $\quad$ Also, $u_b=\dfrac{\omega_b}{\omega_n}$

$$\therefore \boxed{\text{Bandwidth, } \omega_b=\omega_n\, u_b=\omega_n\left[1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4}\right]^{\frac{1}{2}}}$$

FREQUENCY RESPONSE PLOTS

Frequency response analysis of control systems can be carried either analytically or graphically. The various graphical techniques available for frequency response analysis are,

1. Bode plot
2. Polar plot
3. Nichols plot
4. M and N circles
5. Nichols chart

The Bode plot, Polar plot and Nichols plot are usually drawn for open loop systems. From the open loop response plot, the performance and stability of closed loop system are estimated. The M and N circles and Nichols chart are used to graphically determine the frequency response of unity feedback closed loop system from the knowledge of open loop response.

The frequency response plots are used to determine the frequency domain specifications, to study the stability of the systems and to adjust the gain of the system to satisfy the desired specifications.

BODE PLOT

The Bode plot is a frequency response plot of the sinusoidal transfer function of a system. A Bode plot consists of two graphs. One is a plot of the magnitude of a sinusoidal transfer function versus log ω. The other is a plot of the phase angle of a sinusoidal transfer function versus log ω.

Sketch Bode plot for the following transfer function and determine the system gain K for the gain cross over frequency to be 5 rad/sec.

$$G(s) = \frac{Ks^2}{(1+0.2s)\,(1+0.02s)}$$

SOLUTION

The sinusoidal transfer function G(jω) is obtained by replacing s by jω in the given s-domain transfer function.

$$\therefore\; G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

Let K = 1, $\therefore\; G(j\omega) = \dfrac{(j\omega)^2}{(1+j0.2\omega)(1+j0.02\omega)}$

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1} = \dfrac{1}{0.2} = 5$ rad / sec and $\omega_{c2} = \dfrac{1}{0.02} = 50$ rad / sec

The various terms of $G(j\omega)$ are listed in Table-1

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$(j\omega)^2$	–	+40	
$\frac{1}{1+j0.2\omega}$	$\omega_{c1}=\frac{1}{0.2}=5$	–20	$40-20=20$
$\frac{1}{1+j0.02\omega}$	$\omega_{c2}=\frac{1}{0.02}=50$	20	$20-20=0$

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

Let, $\omega_l = 0.5$ rad/sec and ω_h=100 rad/sec.

Let, $A = |G(j\omega)|$ in db.

Let us calculate A at $\omega_l, \omega_{c1}, \omega_{c2}$ and ω_h.

$$\text{At } \omega = \omega_l, \quad A = 20\log\left|(j\omega)^2\right| = 20\log(\omega)^2 = 20\log(0.5)^2 = -12 \text{ db}$$

$$\text{At } \omega = \omega_{c1}, \quad A = 20\log\left|(j\omega)^2\right| = 20\log(\omega)^2 = 20\log(5)^2 = 28 \text{ db}$$

$$\text{At } \omega = \omega_{c2}, \quad A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega = \omega_{c1})} = 20 \times \log\frac{50}{5} + 28 = 48 \text{ db}$$

$$\text{At } \omega = \omega_h, \quad A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A_{(\text{at } \omega = \omega_{c2})} = 0 \times \log\frac{100}{50} + 48 = 48 \text{ db}$$

Let the points a, b, c and d be the points corresponding to frequencies $\omega_l, \omega_{c1}, \omega_{c2}$ and ω_h respectively on the magnitude plot.

PHASE PLOT

The phase angle of $G(j\omega)$ as a function of ω is given by,

$$\phi = \angle G(j\omega) = 180^\circ - \tan^{-1}0.2\omega - \tan^{-1}0.02\omega$$

The phase angle of $G(j\omega)$ are calculated for various values of ω and listed in table-2.

TABLE-2

ω rad/sec	$\tan^{-1} 0.2\omega$ deg	$\tan^{-1} 0.02\omega$ deg	$\phi = \angle G(j\omega)$ deg
0.5	5.7	0.6	$173.7 \approx 174$
1	11.3	1.1	$167.6 \approx 168$
5	45	5.7	$129.3 \approx 130$
10	63.4	11.3	$105.3 \approx 106$
50	84.3	45	$50.7 \approx 50$
100	87.1	63.4	$29.5 \approx 30$

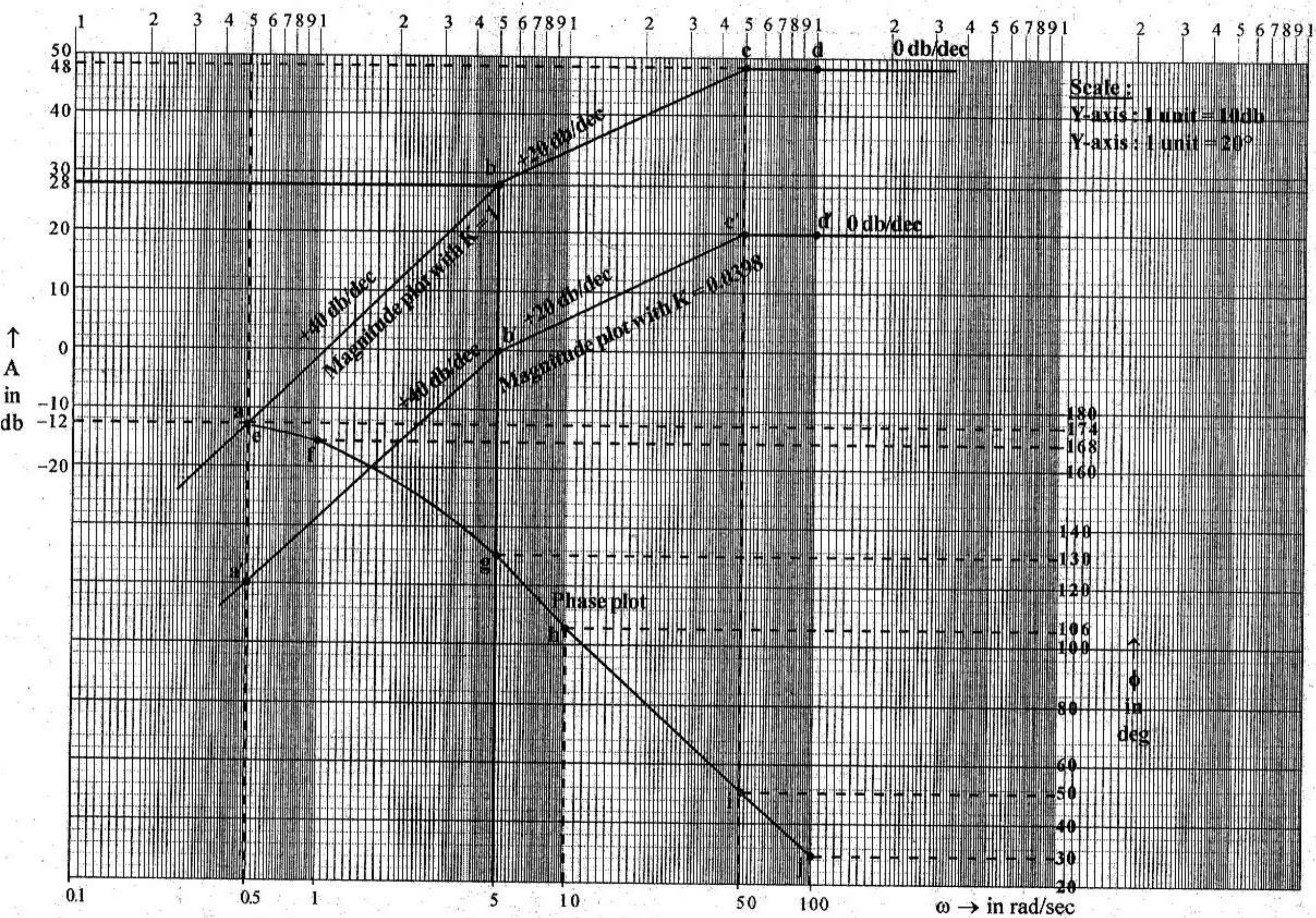


Fig 3.1.1 : Bode plot of transfer function ... $K(j\omega)^2$

CALCULATION OF K

Given that the gain crossover frequency is 5 rad/sec. At $\omega = 5$ rad/sec the gain is 28 db. If gain crossover frequency is 5 rad/sec then at that frequency the db gain should be zero. Hence to every point of magnitude plot a db gain of –28db should be added. The addition of –28db shifts the plot downwards. The corrected magnitude plot is obtained by shifting the plot with K = 1 by –28db downwards. The magnitude correction is independent of frequency. Hence the magnitude of –28db is contributed by the term K. The value of K is calculated by equating 20 logK to –28 db.

$$\therefore 20 \log K = -28 \text{ db}$$

$$\log K = \frac{-28}{20} \; ; \; K = 10^{-\left(\frac{28}{20}\right)} = 0.0398$$

Sketch the bode plot for the following transfer function and determine phase margin and gain margin.

$$G(s) = \frac{75\,(1+0.2s)}{s\left(s^2+16s+100\right)}$$

SOLUTION

On comparing the quadratic factor in the denominator of G(s) with standard form of quadratic factor we can estimate ζ and ω_n.

$$\therefore s^2+16s+100 = s^2+2\zeta\omega_n s+\omega_n^2$$

On comparing we get,

$$\omega_n^2 = 100 \quad \Rightarrow \quad \omega_n = 10$$

$$2\zeta\omega_n = 16 \quad \Rightarrow \quad \zeta = \frac{16}{2\omega_n} = \frac{16}{2\times 10} = 0.8$$

Let us convert the given s-domain transfer function into bode form or time constant form.

$$\therefore G(s) = \frac{75(1+0.2s)}{s\left(s^2+16s+100\right)} = \frac{75\,(1+0.2s)}{s\times 100\left(\frac{s^2}{100}+\frac{16s}{100}+1\right)} = \frac{0.75\,(1+0.2s)}{s\left(1+0.01s^2+0.16s\right)}$$

Let us convert the given s-domain transfer function into bode form or time constant form.

$$\therefore G(s) = \frac{75(1+0.2s)}{s\left(s^2+16s+100\right)} = \frac{75\,(1+0.2s)}{s\times 100\left(\frac{s^2}{100}+\frac{16s}{100}+1\right)} = \frac{0.75\,(1+0.2s)}{s\left(1+0.01s^2+0.16s\right)}$$

The sinusoidal transfer function G(jω) is obtained by replacing s by jω in G(s).

$$\therefore G(j\omega) = \frac{0.75(1+0.2j\omega)}{j\omega\left(1+0.01(j\omega)^2+0.16j\omega\right)} = \frac{0.75(1+j0.2\omega)}{j\omega\left(1-0.01\omega^2+j0.16\omega\right)}$$

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1} = \frac{1}{0.2} = 5 \text{ rad/sec}$ and $\omega_{c2} = \omega_n = 10 \text{ rad/sec}$

Note : For the quadratic factor the corner frequency is ω_n.

The various terms of G(jω) are listed in table-1 in the increasing order of their corner frequencies. Also the table the slope contributed by each term and the change in slope at the corner frequency.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{0.75}{j\omega}$	–	–20	
$1+j0.2\omega$	$\omega_{c1} = \frac{1}{0.2} = 5$	20	$-20+20=0$
$\frac{1}{1-0.01\omega^2+j0.16\omega}$	$\omega_{c2} = \omega_n = 10$	– 40	$0-40=-40$

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

Let, $\omega_l = 0.5$ rad/sec and $\omega_h = 20$ rad/sec.

Let, $A = |G(j\omega)|$ in db .

Let us calculate A at ω_l, ω_{c1}, ω_{c2} and ω_h.

$$\text{At,}\quad \omega = \omega_l,\quad A = 20\log\left|\frac{0.75}{j\omega}\right| = 20\log\frac{0.75}{0.5} = 3.5\text{ db}$$

$$\text{At,}\quad \omega = \omega_{c1},\quad A = 20\log\left|\frac{0.75}{j\omega}\right| = 20\log\frac{0.75}{5} = -16.5\text{ db}$$

$$\text{At,}\quad \omega = \omega_{c2},\quad A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega = \omega_{c1})}$$

$$= 0 \times \log\frac{10}{5} + (-16.5) = -16.5\text{ db}$$

$$\text{At}\quad \omega = \omega_h,\quad A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A_{(\text{at } \omega = \omega_{c2})}$$

$$= -40 \times \log\frac{20}{10} + (-16.5) = -28.5\text{ db}$$

Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot.

PHASE PLOT

The phase angle of $G(j\omega)$ as a function of ω is given by,

$$\phi = \angle G(j\omega) = \tan^{-1}0.2\omega - 90^\circ - \tan^{-1}\frac{0.16\omega}{1-0.01\omega^2} \quad \text{for } \omega \le \omega_n$$

$$\phi = \angle G(j\omega) = \tan^{-1}0.2\omega - 90^\circ - \left(\tan^{-1}\frac{0.16\omega}{1-0.01\omega^2} + 180^\circ\right) \quad \text{for } \omega > \omega_n$$

> ***Note :*** *In quadratic factors the phase varies from 0° to 180°. But calculator calculates* $\tan^{-1}$ *only between 0° to 90°. Hence a correction of 180° should be added to phase after* ω_n.

The phase angle of G(jω) are calculated for various values of w and listed in Table-2.

TABLE-2

ω rad/sec	$-0.2\ \omega\ (180^0/\pi)$ deg	$\tan^{-1} 0.5\ \omega$ deg	$\tan^{-1} 0.125\ \omega$ deg	$\phi = \angle G(j\omega)$ deg
0.01	−0.1145	0.2864	0.0716	−90.4 ≈ −90
0.1	−1.145	2.862	0.716	−94.7 ≈ −94
0.5	−5.7	14	3.6	−113.3 ≈ −114
1	−11.4	26	7.12	−134.4 ≈ −134
2	−22.9	45	14	−171.9 ≈ −172
3	−34.37	56.30	20.56	−201.2 ≈ −202
4	−45.84	63.43	26.57	−225.8 ≈ −226

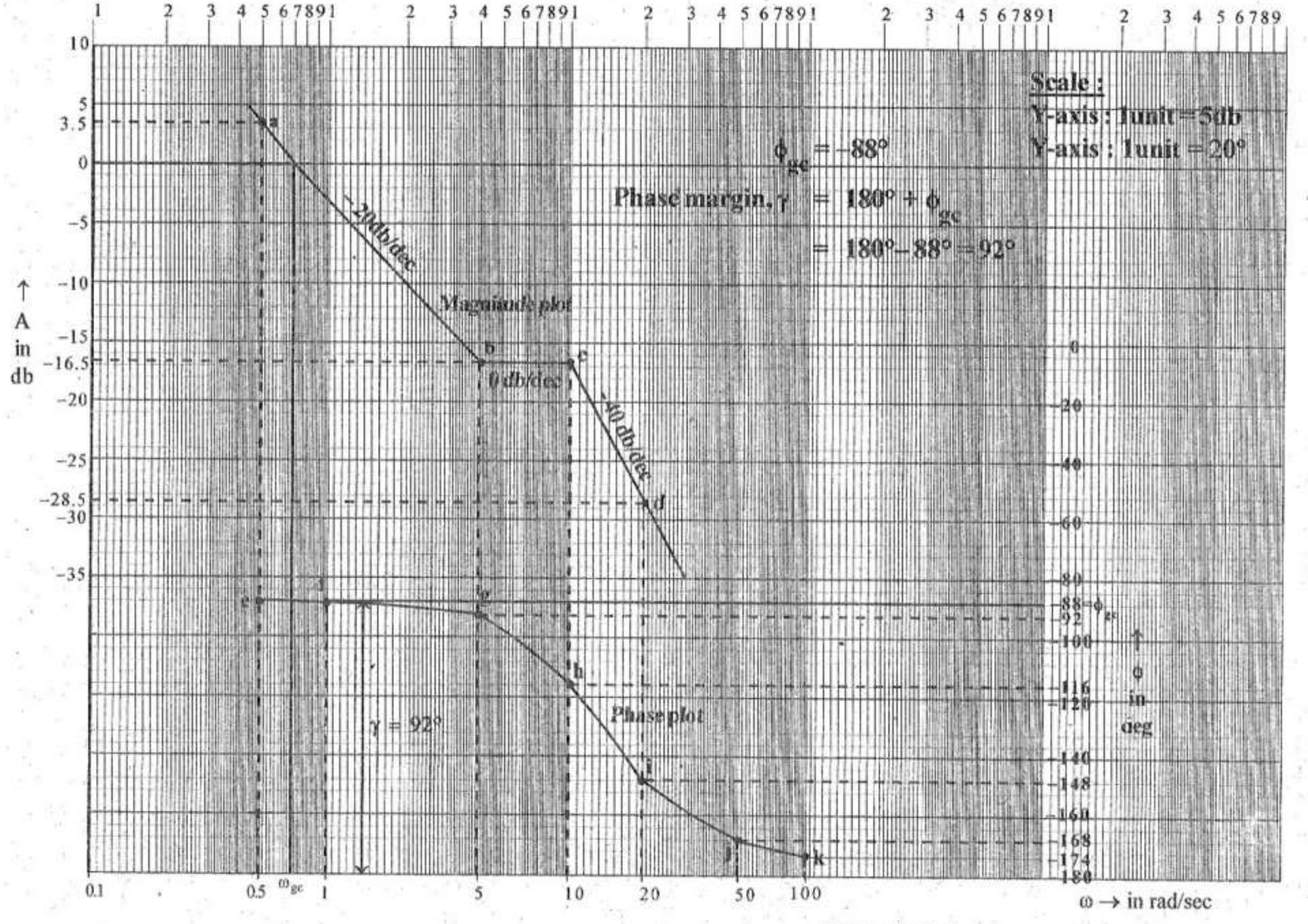


Fig 3.2.1 : Bode plot of transfer function $G(j\omega) = \frac{0.75(1+j0.2\omega)}{\ldots}$

Let ϕ_{gc} be the phase of G(jω) at gain cross-over frequency, ω_{gc}.

From the fig 3.2.1, we get, $\phi_{gc} = 88°$

$$\therefore \text{Phase margin, } g = 180° + \phi_{gc} = 180° - 88° = 92°$$

The phase plot crosses $-180°$ only at infinity. The $|G(j\omega)|$ at infinity is $-\infty$ db.

Hence gain margin is $+\infty$.

The open loop transfer function of a unity feedback system is given by G(s) = 1/s(1+s) (1+2s). Sketch the polar plot and determine the gain margin and phase margin.

SOLUTION

Given that, G(s) = 1/s(1+s) (1 +2s)

Put $s = j\omega$.

$$\therefore G(j\omega) = \frac{1}{j\omega\,(1+j\omega)\,(1+j2\omega)}$$

The corner frequencies are $\omega_{c1} = 1/2 = 0.5$ rad/sec and $\omega_{c2} = 1$ rad/sec. The magnitude and phase angle of $G(j\omega)$ are calculated for the corner frequencies and for frequencies around corner frequencies and tabulated in table-1. Using polar to rectangular conversion, the polar coordinates listed in table-1 are converted to rectangular coordinates and tabulated in table-2.

$$G(j\omega) = \frac{1}{(j\omega)(1+j\omega)(1+j2\omega)} = \frac{1}{\omega\angle 90^\circ \sqrt{1+\omega^2}\angle \tan^{-1}\omega \sqrt{1+4\omega^2}\angle \tan^{-1}2\omega}$$

$$= \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}} \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

$$\therefore |G(j\omega)| = \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}} = \frac{1}{\omega\sqrt{1+4\omega^2+\omega^2+4\omega^4}} = \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

TABLE-1 : Magnitude and phase of G(jω) at various frequencies

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
|G(jω)|	2.2	1.8	1.5	1.2	0.9	0.7	0.3
∠G(jω) deg	-144	-150	-156	-162	-171	-179.5 ≈ -180.	-198

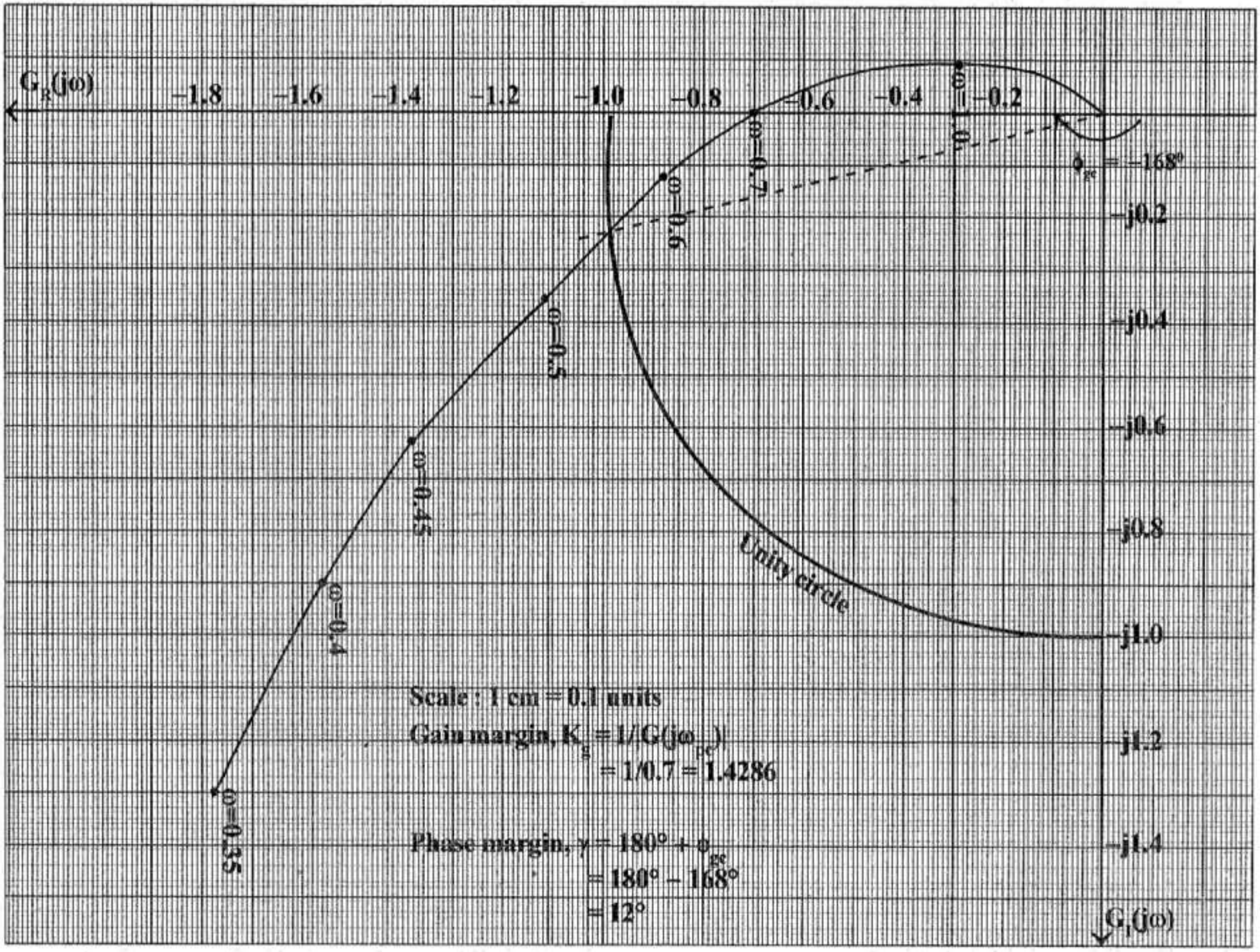
TABLE-2 : Real and imaginary part of $G(j\omega)$ at various frequencies

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
$G_R(j\omega)$	-1.78	-1.56	-1.37	-1.14	-0.89	-0.7	-0.29
$G_I(j\omega)$	-1.29	-0.9	-0.61	-0.37	-0.14	0	0.09

RESULT

Gain margin, $K_g = 1.4286$

Phase margin, $\gamma = +12^{\circ}$



G_R(jω)
−1.8
−1.6
−1.4
−1.2
−1.0
−0.8
−0.6
−0.4
−0.2
ω=1.0
ω=0.7
ω=0.6
ω=0.5
ω=0.45
ω=0.4
ω=0.35
φ_gc = −168°
−j0.2
−j0.4
−j0.6
−j0.8
−j1.0
−j1.2
−j1.4
Unity circle
Scale : 1 cm = 0.1 units
Gain margin, K_g = 1/|G(jω_pc)|
= 1/0.7 = 1.4286
Phase margin, γ = 180° + φ_gc
= 180° − 168°
= 12°
G_I(jω)

Refer control systems by A.Nagoorkani for remaining problems